入学年度	学部	学科	組	番号	検	フリガナ
						氏名

1 p, q を 0 でない相異なる 2 つの実定数とし，$A = \begin{pmatrix} p+q & -pq \\ 1 & 0 \end{pmatrix}$ とおく．

a) ベクトル $\begin{pmatrix} \frac{p}{p-q} \\ \frac{1}{p-q} \end{pmatrix}$，$\begin{pmatrix} \frac{q}{q-p} \\ \frac{1}{q-p} \end{pmatrix}$ はともに A の固有ベクトルであることを示せ．また，それぞれの固有値を求めよ．

b) $P = \begin{pmatrix} \frac{p}{p-q} & \frac{q}{q-p} \\ \frac{1}{p-q} & \frac{1}{q-p} \end{pmatrix} = \frac{1}{p-q}\begin{pmatrix} p & -q \\ 1 & -1 \end{pmatrix}$ とするとき，$P^{-1}AP$ を計算せよ．

c) 数列 $(a_n)_{n\in\mathbf{N}}$，$(b_n)_{n\in\mathbf{N}}$ は連立漸化式 $\begin{pmatrix} b_{n+1} \\ a_{n+1} \end{pmatrix} = \begin{pmatrix} p+q & -pq \\ 1 & 0 \end{pmatrix}\begin{pmatrix} b_n \\ a_n \end{pmatrix}$ をみたすとする．このとき，$\begin{pmatrix} d_n \\ c_n \end{pmatrix} = P^{-1}\begin{pmatrix} b_n \\ a_n \end{pmatrix}$ とおくと，ある行列 B を用いて $\begin{pmatrix} d_{n+1} \\ c_{n+1} \end{pmatrix} = B\begin{pmatrix} d_n \\ c_n \end{pmatrix}$ と書ける．B を求めよ．

d) c_n，d_n をそれぞれ，c_1，d_1 と n を用いて表せ．

e) a_n を a_1，a_2 と n を用いて表せ．

【裏に続く】

2 On planet Xsldkfjs, there are two primitive life forms, the Fleoifkjs and the Wowopfkjs. The Fleoifkjs reproduce wildly, while the Wowopfkjs are essentially parasites on the Fleoifkjs, needing their bodies to reproduce and then killing the host Fleoifkjs. If we let f_k and w_k be the number of Fleoifkjs and Wowopfkjs at the end of year k, respectively. Then their populations are governed by the rule

$$\begin{pmatrix} f_{k+1} \\ w_{k+1} \end{pmatrix} = \begin{pmatrix} \frac{13}{8} & -\frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} \end{pmatrix} \begin{pmatrix} f_k \\ w_k \end{pmatrix}.$$

a) Let $A = \begin{pmatrix} \frac{13}{8} & -\frac{3}{8} \\ \frac{3}{8} & \frac{3}{8} \end{pmatrix}$. Find the eigenvalues and eigenvectors of A.

b) Find a matrix P such that $P^{-1}AP$ is a diagonal matrix

c) Show that in the long term, both populations are growing. Determine the growth rate and the eventual ratio of Fleoifkjs to Wowopfkjs.

d) If there are few Fleoifkjs and too many of the parasitic Wowopfkjs around initially, both populations will perish. Find this critical ratio of Fleoifkjs to Wowopfkjs below which both populations are doomed to oblivion.

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