8 Logarithms (cont.)

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1 Simplify each of the following

a) $\sqrt{2} \times \sqrt[3]{4} \div \sqrt[6]{2} =$ $2^{\frac{1}{2}} \times (2^2)^{\frac{1}{3}} \times 2^{-\frac{1}{6}}$

$= 2^{\frac{3}{6}+\frac{4}{6}-\frac{1}{6}} = 2$

b) $2^{\frac{1}{3}} \div 4^{\frac{1}{4}} \times 32^{-\frac{1}{6}} =$ $2^{\frac{1}{3}} \times 2^{-\frac{1}{2}} \times (2^5)^{-\frac{1}{6}}$

$= 2^{\frac{2}{6}-\frac{3}{6}-\frac{5}{6}} = 2^{-1} = \frac{1}{2}$

c) $\sqrt[3]{a^2} \times \sqrt[4]{a} \div \sqrt[6]{a\sqrt{a}} =$ $a^{\frac{2}{3}} \times a^{\frac{1}{4}} \times (a \times a^{\frac{1}{2}})^{-\frac{1}{6}}$

$= a^{\frac{8}{12}+\frac{3}{12}-\frac{3}{12}} = a^{\frac{2}{3}}$

d) $\log_4 12 + \log_4 32 - \log_4 6$

$= \log_4 \frac{2^2 \times 3 \times 2^5}{2 \times 3} = \log_4 2^6 = \log_4 4^3 = 3$

e) $\frac{2}{3}\log_3 8 + 2\log_3 \sqrt{5} - \log_3 180$

$= \log_3 \frac{(2^3)^{\frac{2}{3}} \times (5^{\frac{1}{2}})^2}{2^2 \times 3^2 \times 5} = \log_3 3^{-2} = -2$

f) $4\log_8 \sqrt{2} + \frac{1}{2}\log_8 3 - \log_8 \frac{\sqrt{3}}{2}$

$= \log_8 \frac{(2^{\frac{1}{2}})^4 \times 3^{\frac{1}{2}}}{\frac{3^{\frac{1}{2}}}{2}} = \log_8 2^3 = \log_8 8 = 1$

2 Assuming $\log_{10} 2 = a$ and $\log_{10} 3 = b$, express each of the following in terms of a and b.

a) $\log_{10} 72 =$ $\log_{10} 2^3 \cdot 3^2 = 3\log_{10} 2 + 2\log_{10} 3$

$= 3a + 2b$

b) $\log_{10} 1.5 = \log_{10} \frac{3}{2} = \log_{10} 3 - \log_{10} 2 = b - a$

c) $\log_3 4 = \frac{\log_{10} 4}{\log_{10} 3} = \frac{2\log_{10} 2}{\log_{10} 3} = \frac{2a}{b}$

3 Arrange each of the following three numbers in ascending order.

a) -1, $\log_2 0.25$, $\log_2 \frac{1}{3}$.

$-1 = \log_2 0.5$ $\qquad \log_2 0.33\ldots$

$\log_2 0.25 < \log_2 \frac{1}{3} < -1$

b) 1, $\log_{\frac{1}{2}} 5$, $\log_{\frac{1}{2}} \frac{1}{3}$

$1 = \log_{\frac{1}{2}} \frac{1}{2}$ $\qquad$ base $\frac{1}{2}$ is less than 1

$\log_{\frac{1}{2}} a < \log_{\frac{1}{2}} b \Leftrightarrow a > b$

$\therefore \log_{\frac{1}{2}} 5 < 1 < \log_{\frac{1}{2}} \frac{1}{3}$

4 Solve each of the following equations for x

a) $4^x = 8\sqrt{2}$

$2^{2x} = 2^3 \times 2^{\frac{1}{2}}$

$2x = 3 + \frac{1}{2} \Rightarrow x = \frac{7}{4}$

b) $\log_2(5-x) = \log_2(x^2-1)$

$5-x > 0,\ x^2-1 > 0,\ 5-x = x^2-1$

$x^2+x-6 = 0 \Rightarrow (x-2)(x+3) = 0 \Rightarrow x = 2, -3$

Both $2, -3$ satisfy $5-x>0,\ x^2-1>0$

$\therefore x = 2, -3$

c) $27^x \geqq 3(\sqrt{3})^x$

$(3^3)^x \geqq 3 \cdot (3^{\frac{1}{2}})^x \Rightarrow 3^{3x} \geqq 3^{\frac{1}{2}x+1}$

$\Rightarrow 3x \geqq \frac{1}{2}x + 1 \Rightarrow x \geqq \frac{2}{5}$

d) $\log_2(3x-1) < 3$

$\log_2(3x-1) < \log_2 2^3$

$\Rightarrow 0 < 3x-1 < 8 \Rightarrow \frac{1}{3} < x < 3$

e) $\log_6 x + \log_6(x-1) \leqq 1$

$x > 0,\ x-1 > 0,\quad x(x-1) \leqq 6$

$x^2 - x - 6 \leqq 0 \Leftrightarrow (x+2)(x-3) \leqq 0$

$1 < x \leqq 3$

5 At which place the first non-zero number appears in 0.6^{30}. Use $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$ if necessary.

$\log_{10} 0.6^{30} = 30\log_{10} \frac{6}{10} = 30(\log_{10} 2 + \log_{10} 3 - 1)$

$\fallingdotseq -6.657$

$10^{-6} > 0.6^{30} > 10^{-7}$

$0.000001 > 0.6^{30} > 0.0000001$ $\qquad \therefore 7^{th}$ place

(0.000001: 6^{th}; 0.0000001: 7^{th})

6 As a clearance sale, a store decided to sell products that did not sell on that day for an additional 10% OFF on the next day. How many days does the price of a product fall below $\frac{1}{3}$ of the original when it remains unsold? Use $\log_{10} 3 = 0.4771$ if necessary.

$0.9^x \leqq \frac{1}{3}$

$\log_{10} 0.9^x \leqq \log_{10} \frac{1}{3}$

$x(2\log_{10} 3 - 1) \leqq -\log_{10} 3$

$x \times (-0.0458) \leqq -0.4771$

$x \geqq \frac{-0.4771}{-0.0458} = 10.417$

$\therefore 11^{th}$ day.