

7 Logarithms

Student ID No.								Name	
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1 Find each of the following values.

a) $\log_3 9 = 2$

b) $\log_2 16 = 4$

c) $\log_4 8 = \log_4 4^{\frac{3}{2}} = \frac{3}{2}$

d) $\log_{\frac{1}{2}} 16 = \log_{\frac{1}{2}} \left(\frac{1}{2}\right)^{-4} = -4$

e) $\log_{10} 0.1 = -1$

f) $\log_9 \sqrt{3} = \log_9 \left(9^{\frac{1}{2}}\right)^{\frac{1}{2}} = \frac{1}{4}$

g) $\log_2 \sqrt[3]{2} = \frac{1}{3}$

h) $\log_{\sqrt{5}} 25 = \log_{\sqrt{5}} \left((\sqrt{5})^2\right)^2 = 4$

i) $\log_{16} \frac{1}{64} = \log_{16} 16^{-\frac{3}{2}} = -\frac{3}{2}$

2 Solve each of the following equations for x

a) $\log_2 x = 3 \quad x = 8$

b) $\log_9 x = \frac{3}{2} \quad x = 9^{\frac{3}{2}} = 27$

c) $\log_2 x = -2 \quad x = \frac{1}{4}$

d) $\log_x 27 = 3 \quad x^3 = 27 \Rightarrow x = 3$

3 Simplify the following.

a) $\log_2 4 + \log_4 8 = 2 + \frac{3}{2} = \frac{7}{2}$

b) $\log_3 2 - \log_3 18 = \log_3 \frac{2}{18} = -2$

c) $\log_3 4 + \log_3 18 - 3 \log_3 2 = \log_3 \frac{4 \times 18}{2^3} = \log_3 9 = 2$

d) $\log_2 \sqrt[3]{12} - \frac{1}{3} \log_2 3 = \log_2 \frac{12^{\frac{1}{3}}}{3^{\frac{1}{3}}} = \log_2 2^{\frac{2}{3}} = \frac{2}{3}$

4 Let $p = \log_a 2$ and $q = \log_a 3$. Express each of the following in terms of p and q .

a) $\log_a 72 = \log_a 2^3 \cdot 3^2 = 3p + 2q$

b) $\log_a \frac{3}{8} = \log_a 3 \cdot 2^{-3} = q - 3p$

c) $\log_a \sqrt[3]{6} = \log_a 2^{\frac{1}{3}} \cdot 3^{\frac{1}{3}} = \frac{1}{3}p + \frac{1}{3}q$

5 Simplify the following using the change-of-base formula.

a) $\log_4 8 = \frac{\log_2 8}{\log_2 4} = \frac{3}{2}$

b) $\log_9 3 = \frac{\log_3 3}{\log_3 9} = \frac{1}{2}$

c) $\log_3 2 \cdot \log_2 27 = \log_3 2 \cdot \frac{\log_3 27}{\log_3 2} = 3$

6 Simplify the following.

a) $\frac{1}{2} \log_5 3 + 3 \log_5 \sqrt{2} - \log_5 \sqrt{24} = \log_5 \frac{3^{\frac{1}{2}} \times 2^{\frac{3}{2}}}{3^{\frac{1}{2}} \times 2^{\frac{3}{2}}} = \log_5 1 = 0$

b) $(\log_2 3 + \log_4 9)(\log_3 4 + \log_9 2) =$
 $= \left(\log_2 3 + \frac{\log_2 9}{\log_2 4}\right) \left(\frac{\log_2 4}{\log_2 3} + \frac{\log_2 2}{\log_2 9}\right)$
 $= 2 \log_2 3 \times \left(\frac{2}{\log_2 3} + \frac{1}{2 \log_2 3}\right)$
 $= 2 \times \left(2 + \frac{1}{2}\right) = 5$