

6 Exponents and radicals

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1 Simplify the following.

a) $2^{-2} = \frac{1}{4}$

b) $(-4)^0 = 1$

c) $(-2)^{-3} = -\frac{1}{8}$

d) $0.1^{-2} = 100$

2 Simplify the following, assuming $a > 0$ and $b > 0$.

a) $a^{-2} \times a^5 = a^3$

b) $a^{-5} \div a^3 = a^{-8}$

c) $(a^{-2})^{-1} = a^2$

d) $(ab^{-1})^{-3} = a^{-3} b^3$

3 Simplify the following.

a) $\sqrt[3]{-64} = -4$

b) $\sqrt[3]{-0.001} = -0.1$

c) $\sqrt[4]{2} \sqrt[4]{8} = 2$

d) $\sqrt[3]{0.01} \times \sqrt[3]{0.1} = 0.1$

e) $\sqrt[3]{375} \div \sqrt[3]{3} = 5$

f) $\sqrt[4]{256} = 125$

g) $\sqrt[3]{\sqrt[4]{2^{12}}} = 2$

4 Simplify each of the following, and express it with a rational exponent. Here, we assume $a > 0$.

a) $\sqrt[3]{a} = a^{\frac{1}{3}}$

b) $(\sqrt{a})^5 = a^{\frac{5}{2}}$

c) $\sqrt[4]{a^5} = a^{\frac{5}{4}}$

d) $\frac{1}{(\sqrt[5]{a})^3} = a^{-\frac{3}{5}}$

5 Simplify the following.

a) $\left(\frac{1}{9}\right)^{-1.5} = 9^{\frac{3}{2}} = 27$

b) $(2^{-2})^{1.5} = 2^{-3} = \frac{1}{8}$

c) $(9^{\frac{5}{3}})^{\frac{9}{10}} = 9^{\frac{3}{2}} = 27$

d) $27^{-\frac{2}{3}} \times 9^{\frac{1}{2}} = (3^3)^{-\frac{2}{3}} \times (3^2)^{\frac{1}{2}} = 3^{-2} \times 3 = \frac{1}{3}$

e) $4^{\frac{1}{2}} \times 8^{\frac{1}{3}} \times 8^{-\frac{1}{2}} = (2^2)^{\frac{1}{2}} \times (2^3)^{\frac{1}{3}} \times (2^3)^{-\frac{1}{2}} = 2^{\frac{1}{2}} = \sqrt{2}$

f) $\left(\frac{1}{2}\right)^{-\frac{3}{4}} \div \left(\frac{1}{2}\right)^{-\frac{1}{4}} = \left(\frac{1}{2}\right)^{-\frac{3}{4} + \frac{1}{4}} = 2^{\frac{1}{2}} = \sqrt{2}$

6 Simplify the following, assuming $a > 0$ and $b > 0$.

a) $(a^{\frac{3}{2}} a^{-1})^4 = a^6 \cdot a^{-4} = a^2$

b) $a^{\frac{1}{4}} \div a^{-\frac{2}{3}} = a^{\frac{1}{4} + \frac{2}{3}} = a^{\frac{11}{12}}$

c) $(8a^{\frac{1}{2}})^{\frac{2}{3}} \times a^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} a^{\frac{1}{2} \times \frac{2}{3} + \frac{2}{3}} = 4a$

d) $(9a^{\frac{2}{3}} b^{-2})^{\frac{1}{2}} = 3 a^{\frac{1}{3}} b^{-1} = \frac{3\sqrt{a}}{b}$

e) $(a^{-\frac{3}{4}})^{-\frac{2}{3}} \div a^{\frac{3}{2}} = a^{\frac{1}{2} - \frac{3}{2}} = a^{-1} = \frac{1}{a}$

7 Simplify the following, assuming $a > 0$ and $b > 0$.

a) $(a^{\frac{1}{2}} - a^{-\frac{1}{2}})^2 = a - 2 \cdot a^{\frac{1}{2}} \cdot a^{-\frac{1}{2}} + a^{-1}$
 $= a - 2 + a^{-1} = a - 2 + \frac{1}{a}$

b) $(a^{\frac{1}{3}} - a^{-\frac{1}{3}})(a^{\frac{2}{3}} + 1 + a^{-\frac{1}{3}}) = (a^{\frac{1}{3}})^3 - (a^{-\frac{1}{3}})^3$
 $= a + a^{\frac{1}{3}} + 1 - a^{\frac{1}{3}} - a^{-\frac{1}{3}} - a^{-\frac{2}{3}} = a - a^{-1} = a - \frac{1}{a}$

c) $(a - b) \div (a^{\frac{1}{3}} - b^{\frac{1}{3}}) = ((a^{\frac{1}{3}})^3 - (b^{\frac{1}{3}})^3) \div (a^{\frac{1}{3}} - b^{\frac{1}{3}})$
 $= a^{\frac{2}{3}} + a^{\frac{1}{3}} \cdot b^{\frac{1}{3}} + b^{\frac{2}{3}}$