

# 5 Quadratic functions

| Student ID No. |   |   |   |   |  |  |  |  |  | Name |  |
|----------------|---|---|---|---|--|--|--|--|--|------|--|
| 1              | 9 | F | 1 | 1 |  |  |  |  |  |      |  |

1 For each of the following parabolas, find the vertex.

a)  $y = -2x^2 + 8x - 5$

$$= -2(x-2)^2 + 3$$

(2, 3)

b)  $y = 3x^2 + 2x + 1$

$$= 3(x + \frac{1}{3})^2 + \frac{2}{3}$$

(-\frac{1}{3}, \frac{2}{3})

c)  $y = 4 - 2x + \frac{1}{2}x^2$

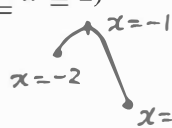
$$= \frac{1}{2}(x-2)^2 + 2$$

(2, 2)

2 For each of the following functions, find the maximum and minimum values when  $x$  varies within the domain indicated in ( ). Also, find the value of  $x$  at which the function attains its maximum and minimum.

a)  $y = -x^2 - 2x - 5$  ( $-2 \leq x \leq 2$ )

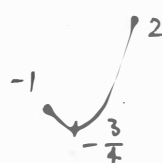
$$= -(x+1)^2 - 4$$



max: -4  
at  $x = -1$   
min: -13  
at  $x = 2$

b)  $y = 2x^2 + 3x - 1$  ( $-1 \leq x \leq 2$ )

$$= 2(x + \frac{3}{4})^2 - \frac{17}{8}$$



max: 13 at  $x = 2$   
min: -\frac{17}{8} at  $x = -\frac{3}{4}$

3 Solve the following equations over the complex numbers.

a)  $2x^2 + 3x - 1 = 0$

$$x = \frac{-3 \pm \sqrt{17}}{4}$$

b)  $3x^2 - 5x + 9 = 0$

$$x = \frac{5 \pm \sqrt{83}i}{6}$$

c)  $x^2 - 2x + 2 = 0$

$$x = 1 \pm i$$

d)  $4x(x + 5) = -25$

$$4x^2 + 20x + 25 = 0$$

$$(2x+5)^2 = 0$$

$x = -\frac{5}{2}$  (double sol.)

e)  $\frac{x^2}{2} + \frac{2x}{3} + \frac{1}{3} = 0$

$$3x^2 + 4x + 2 = 0$$

$$x = \frac{-2 \pm \sqrt{2}i}{3}$$

4 Solve the following inequalities.

a)  $2x^2 + x - 6 > 0$

$$(2x-3)(x+2) > 0$$

$$x < -2, x > \frac{3}{2}$$

b)  $2x^2 - 3x - 2 \leq 0$

$$(2x+1)(x-2) \leq 0$$

$$-\frac{1}{2} \leq x \leq 2$$

c)  $2(x^2 - x) < 3(x + 1)$

$$2x^2 - 5x - 3 < 0$$

$$(2x+1)(x-3) < 0$$

$$-\frac{1}{2} < x < 3$$

d)  $x^2 + x - 1 < 0$

$$x^2 + x - 1 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{5}}{2}$$

$$\therefore \frac{-1 - \sqrt{5}}{2} < x < \frac{-1 + \sqrt{5}}{2}$$

e)  $4x^2 - 4x + 1 > 0$

$$(2x-1)^2 > 0$$

$$x \neq \frac{1}{2}$$

f)  $2x^2 - 3x + 2 < 0$

$$2(x - \frac{3}{4})^2 + \frac{7}{8} < 0$$

No solution.

5 There is a rectangle whose vertical length is 1cm longer than the horizontal length. When the area is 21cm<sup>2</sup>, what is the length of the vertical side and the horizontal side, respectively.

$x$ : horizontal length.

$$x(x+1) = 21$$

$$x^2 + x - 21 = 0$$

$$x = \frac{-1 \pm \sqrt{85}}{2}$$

$x > 0$  &  $y$

$$x = \frac{\sqrt{85}-1}{2} \dots \text{horizontal}$$

$$\frac{\sqrt{85}+1}{2} \dots \text{vertical}$$

6 A rocket is launched in the air. Its height, in meters above sea level, as a function of time, in seconds, is given by  $h(t) = -4.9t^2 + 147t - 602.5$ . Find the maximum height the rocket attains.

$$h(t) = -4.9(t-15)^2 + 500$$

500 m ( $t=15$ )