3 ❦ Fractions

1. Simplify the following expressions.

a) $\dfrac{6bx}{4a^2x^2} = \dfrac{3b}{2a^2x}$

b) $\dfrac{c}{\dfrac{ab}{ab^2c}} = bc^2$

c) $\left(-\dfrac{2xy^2}{a^2}\right) \div \left(-\dfrac{ay}{4x^2}\right) = \dfrac{2xy^2}{a^2} \times \dfrac{4x^2}{ay} = \dfrac{8x^3y}{a^3}$

d) $\dfrac{3x}{6x^2+x} = \dfrac{3}{6x+1}$

e) $\dfrac{x^2-5x+6}{x+3} \times \dfrac{x^2+3x}{x^2-6x+9}$

$= \dfrac{(x-2)(x-3)}{x+3} \times \dfrac{x(x+3)}{(x-3)^2}$

$= \dfrac{x(x-2)}{x-3}$

f) $\dfrac{x^2+x-6}{x} \div \dfrac{x^2-4}{x^2} = \dfrac{(x-2)(x+3)}{x} \times \dfrac{x^2}{(x-2)(x+2)}$

$= \dfrac{x(x+3)}{x+2}$

g) $\left(1+\dfrac{b}{x}\right) \div \left(\dfrac{a}{x}-1\right) = \dfrac{x+b}{x} \div \dfrac{a-x}{x}$

$= \dfrac{x+b}{x} \times \dfrac{x}{a-x} = \dfrac{x+b}{a-x}$

h) $\dfrac{1}{c-\dfrac{1}{c+\dfrac{1}{c}}} = \dfrac{1}{c-\dfrac{1}{\dfrac{c^2+1}{c}}} = \dfrac{1}{c-\dfrac{c}{c^2+1}}$

$= \dfrac{1}{\dfrac{c^3+c-c}{c^2+1}} = \dfrac{c^2+1}{c^3}$

2. For each of the following pairs of polynomials, factor the polynomials and find the greatest common divisor (GCD) and the least common multiple (LCM).

a) $\begin{cases} x^2+3x+2 = (x+1)(x+2) \\ 2x^2+x-1 = (2x-1)(x+1) \end{cases}$

$\begin{cases} \text{GCD}= (x+1) \\ \text{LCM}= (x+1)(x+2)(2x-1) \end{cases}$

b) $\begin{cases} x^2-x = x(x-1) \\ x^2+2x-3 = (x+3)(x-1) \\ x^2-2x+1 = (x-1)^2 \end{cases}$

$\begin{cases} \text{GCD}= x-1 \\ \text{LCM}= x(x-1)^2(x+3) \end{cases}$

3. Simplify the following expressions.

a) $\dfrac{x+2}{x-2} + \dfrac{4}{2-x} = \dfrac{x+2}{x-2} + \dfrac{-4}{x-2} = \dfrac{x-2}{x-2} = 1$

b) $\dfrac{1}{x+1} + \dfrac{2x}{1-x^2} = \dfrac{1}{x+1} + \dfrac{2x}{(1-x)(1+x)}$

$= \dfrac{1-x+2x}{(1-x)(1+x)} = \dfrac{1+x}{(1-x)(1+x)} = \dfrac{1}{1-x}$

c) $\dfrac{1}{2x^2+3x+1} - \dfrac{2}{2x^2+x-1}$

$= \dfrac{1}{(2x+1)(x+1)} - \dfrac{2}{(2x-1)(x+1)} = \dfrac{2x-1-2(2x+1)}{(2x+1)(2x-1)(x+1)}$

$= \dfrac{-2x-3}{(2x+1)(2x-1)(x+1)}$

d) $\dfrac{a-1}{a} - \dfrac{a}{a-1} + \dfrac{1}{a-1}$

$= \dfrac{(a-1)^2-a^2+a}{a(a-1)} = \dfrac{-a+1}{a(a-1)} = -\dfrac{1}{a}$

e) $\dfrac{1}{a^2-ab} + \dfrac{1}{b^2-ab}$

$= \dfrac{1}{a(a-b)} + \dfrac{1}{b(b-a)} = \dfrac{b-a}{ab(a-b)}$

$= -\dfrac{1}{ab}$

f) $\dfrac{3x-4}{x^2-3x+2} + \dfrac{x-1}{x^2-x-1}$

$= \dfrac{3x-4}{(x-2)(x-1)} + \dfrac{x-1}{x^2-x-1}$

$= \dfrac{(3x-4)(x^2-x-1)+(x-1)^2(x-2)}{(x-2)(x-1)(x^2-x-1)}$

$= \dfrac{4x^3-11x^2+6x+2}{(x-1)(x-2)(x^2-x-1)}$