2 ❧ Factorization and division

		Student ID No.								Name
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1 Factor the following polynomials.

a) $3a^2b + 6ab^2 = 3ab(a+2b)$

b) $x^2 + 7x + 12 = (x+3)(x+4)$

c) $3x^2 + 10x + 8 = (3x+4)(x+2)$

d) $2x^2 - 5xy - 12y^2 = (2x+3y)(x-4y)$

e) $a^3 - 8b^3 = (a-2b)(a^2+2ab+4b^2)$

f) $2x^4 - 16x = 2x(x-2)(x^2+2x+4)$

g) $x^2 + \frac{5}{2}x + 1 = (x+\frac{1}{2})(x+2)$

h) $(x-4)(5x+1) + 16 = (5x-4)(x-3)$

2 Using long division, find the quotient and the remainder. (Here, a is a constant.)

a)

$$
\begin{array}{r}
2x^2+3x+5 \\
2x-3\ \overline{)\ 4x^3\qquad\ +x+1} \\
\underline{4x^3-6x^2}\qquad\qquad \\
6x^2+x \\
\underline{6x^2-9x} \\
10x+1 \\
\underline{10x-15} \\
16
\end{array}
$$

Quotient $= 2x^2+3x+5$

Remainder $= 16$

b)

$$
\begin{array}{r}
2x-3 \\
x^2+2x-3\ \overline{)\ 2x^3+x^2-13x+6} \\
\underline{2x^3+4x^2-6x} \\
-3x^2-7x+6 \\
\underline{-3x^2-6x+9} \\
-x-3
\end{array}
$$

Quotient $= 2x-3$

Remainder $= -x-3$

c)

$$
\begin{array}{r}
x-a \\
x^2+ax-2a^2\ \overline{)\ x^3\qquad\ -3a^2x} \\
\underline{x^3+ax^2-2a^2x} \\
-ax^2-a^2x \\
\underline{-ax^2-a^2x+2a^3} \\
-2a^3
\end{array}
$$

Quotient $= x-a$

Remainder $= -2a^3$

3 a) For the function $f(x) = x^3 - 6x^2 + 11x - 6$, calculate the following values of .

$f(1) = 1-6+11-6 = 0$

$f(2) = 8-24+22-6 = 0$

$f(-1) = -1-6-11-6 = -24$

$f(-2) = -8-24-22-6 = -60$

b) Factor the polynomial $f(x) = x^3 - 6x^2 + 11x - 6$.

Since $f(1) = f(2) = 0$, $f(x)$ is divisible by

$(x-1)(x-2) = x^2-3x+2$.

$f(x) = (x-1)(x-2)(x-3)$

$$
\begin{array}{r}
x-3 \\
x^2-3x+2\ \overline{)\ x^3-6x^2+11x-6} \\
\underline{x^3-3x^2+2x} \\
-3x^2+9x-6 \\
\underline{-3x^2+9x-6} \\
0
\end{array}
$$

4 Let $f(x) = x^3 - 5x^2 + 7x - 3$.

a) Find the value $f(1)$.

$f(1) = 1-5+7-3 = 0$

b) Factor $f(x)$.

$f(x) = (x-1)(x^2-4x+3)$

$= (x-1)(x-1)(x-3)$

$= (x-1)^2(x-3)$

$$
\begin{array}{r}
x^2-4x+3 \\
x-1\ \overline{)\ x^3-5x^2+7x-3} \\
\underline{x^3-x^2} \\
-4x^2+7x-3 \\
\underline{-4x^2+4x} \\
3x-3 \\
\underline{3x-3} \\
0
\end{array}
$$

5 Express the following fractions as the sum of a polynomial and a fraction whose numerator has a degree less than its denominator.

a) $\dfrac{6x+1}{2x-1} = 3 + \dfrac{4}{2x-1}$

$$
\begin{array}{r}
3 \\
2x-1\ \overline{)\ 6x+1} \\
\underline{6x-3} \\
4
\end{array}
$$

b) $\dfrac{6x^3+11x^2-31x+15}{3x-2} = 2x^2+5x-7 + \dfrac{1}{3x-2}$

$$
\begin{array}{r}
2x^2+5x-7 \\
3x-2\ \overline{)\ 6x^3+11x^2-31x+15} \\
\underline{6x^3-4x^2} \\
15x^2-31x \\
\underline{15x^2-10x} \\
-21x+15 \\
\underline{-21x+14} \\
1
\end{array}
$$