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10 Derivatives

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1 Find the derivative of the function $f(x) = x^3 - x$ at $x = -1$ by directly calculating the limit $f'(-1) = \lim_{h\to 0} \frac{f(-1+h) - f(-1)}{h}$.

$$f'(-1) = \lim_{h\to 0} \frac{(-1+h)^3 - (-1+h) - ((-1)^3 - (-1))}{h}$$

$$= \lim_{h\to 0} \frac{-1 + 3h - 3h^2 + h^3 + 1 - h - 0}{h}$$

$$= \lim_{h\to 0} \frac{2h - 3h^2 + h^3}{h}$$

$$= \lim_{h\to 0} (2 - 3h + h^2) = 2$$

2 For the following functions, use the definition of derivative $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ to calculate the derivative of each function.

a) $f(x) = -2x + 1$

$$f'(x) = \lim_{h\to 0} \frac{-2(x+h) + 1 - (-2x+1)}{h}$$

$$= \lim_{h\to 0} \frac{-2x - 2h + 1 + 2x - 1}{h}$$

$$= \lim_{h\to 0} \frac{-2h}{h} = -2$$

b) $f(x) = 2x^2 + 5$

$$f'(x) = \lim_{h\to 0} \frac{2(x+h)^2 + 5 - (2x^2+5)}{h}$$

$$= \lim_{h\to 0} \frac{2x^2 + 4xh + 2h^2 + 5 - 2x^2 - 5}{h}$$

$$= \lim_{h\to 0} (4x + 2h) = 4x$$

c) $f(x) = (1-2x)^2$

$$f'(x) = \lim_{h\to 0} \frac{(1-2(x+h))^2 - (1-2x)^2}{h}$$

$$= \lim_{h\to 0} \frac{1 - 4(x+h) + 4(x+h)^2 - (1 - 4x + 4x^2)}{h}$$

$$= \lim_{h\to 0} \frac{-4h + 8xh + 4h^2}{h}$$

$$= \lim_{h\to 0} (-4 + 8x + 4h) = -4 + 8x$$

3 For the following polynomial functions, find the derivatives.

a) $f(x) = -3x^2 - 7x + 6$.

$$f'(x) = -6x - 7$$

b) $f(x) = 3x^3 + 2x^2 + x - 26$.

$$f'(x) = 9x^2 + 4x + 1$$

c) $f(x) = (x-1)(x^2 + x + 1)$.

$$f(x) = x^3 - 1$$

$$f'(x) = 3x^2$$

4 The revenue generated by selling x items is given by $R(x) = 2x^2 + 10x$.

a) Find the average change of the revenue function as x changes from $x = 10$ to $x = 20$.

$$\frac{R(20) - R(10)}{20 - 10} = \frac{1000 - 300}{10} = 70$$

b) Find $R'(10)$.

$$R'(x) = 4x + 10$$

$$R'(10) = 50$$

c) Find $R'(15)$, and show that it coincides with the answer of a).

$$R'(15) = 4 \times 15 + 10 = 70$$

$R'(15)$ is indeed equal to the answer of a)