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10 Limits, AROC

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1 Evaluate each of the following limits.

a) $\lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{(x-3)(x+3)}{x-3}$

$= \lim_{x\to 3} (x+3) = 6$

b) $\lim_{x\to 2} \frac{x^2-6x+8}{x-2} = \lim_{x\to 2} \frac{(x-2)(x-4)}{x-2}$

$= \lim_{x\to 2} (x-4) = -2$

c) $\lim_{x\to -2} \frac{x^3+8}{x+2} = \lim_{x\to -2} \frac{(x+2)(x^2-2x+4)}{x+2}$

$= \lim_{x\to -2} (x^2-2x+4) = 12$

d) $\lim_{x\to 3} \frac{x^2-9}{x^2-5x+6} = \lim_{x\to 3} \frac{(x-3)(x+3)}{(x-3)(x-2)}$

$= \lim_{x\to 3} \frac{x+3}{x-2} = 6$

e) $\lim_{x\to 4} \frac{\frac{1}{x}-\frac{1}{4}}{x-4} = \lim_{x\to 4} \frac{\frac{4-x}{4x}}{x-4} = \lim_{x\to 4} \left(-\frac{1}{4x}\right)$

$= -\frac{1}{16}$

f) $\lim_{h\to 0} \frac{(3+h)^2-9}{h} = \lim_{h\to 0} \frac{9+6h+h^2-9}{h}$

$= \lim_{h\to 0} (6+h) = 6$

g) $\lim_{h\to 0} \frac{(2-h)^3-8}{h} = \lim_{h\to 0} \frac{8-12h+6h^2-h^3-8}{h}$

$= \lim_{h\to 0} (-12+6h-h^2)$

$= -12$

h) $\lim_{h\to 0} \frac{\frac{1}{a+h}-\frac{1}{a}}{h} = \lim_{h\to 0} \frac{1}{h} \times \frac{a-(a+h)}{(a+h)a}$

$= \lim_{h\to 0} \frac{-h}{h(a+h)a}$

$= -\frac{1}{a^2}$

2 Find the average rate of change of $f(x) = -\frac{2}{x^2}$ from $x = 1$ to $x = 2$.

$\frac{f(2)-f(1)}{2-1} = \frac{-\frac{1}{2}-(-2)}{1} = \frac{3}{2}$

3 Find the average rate of change of $f(x) = \frac{3+x}{2-x}$ from $x = a$ to $x = a+h$, and simplify it as much as possible.

$\frac{f(a+h)-f(a)}{h} = \frac{\frac{3+(a+h)}{2-(a+h)}-\frac{3+a}{2-a}}{h}$

$= \frac{1}{h} \frac{(3+a+h)(2-a)-(3+a)(2-a-h)}{(2-a-h)(2-a)}$

$= \frac{1}{h} \frac{h(2-a)+h(3+a)}{(2-a-h)(2-a)} = \frac{1}{h} \frac{5h}{(2-a-h)(2-a)}$

$= \frac{5}{(2-a-h)(2-a)}$

4 The volume V of a sphere with respect to its radius r is given by $V = \frac{4}{3}\pi r^3$. Find the average rate of change of V as r changes from 1 cm to 2 cm.

$\frac{\frac{4}{3}\pi \times 2^3 - \frac{4}{3}\pi \times 1^3}{2-1} = \frac{4}{3}\pi \times (8-1)$

$= \frac{28\pi}{3}$

5 The height of a projectile is given by $s(t) = -64t^2+192t$. Find the average rate of change of the height from $t = 1$ second to $t = 1.5$ seconds.

$\frac{s(1.5)-s(1)}{1.5-1} = \frac{-64\times 1.5^2+192\times 1.5-(-64+192)}{0.5}$

$= \frac{144-128}{0.5} = 32$