May 22, 2019

Midterm Exam

Student ID No.											Name
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1 a) Simplify the following

$4A-3\big(B-2(C-2(B-A))\big) = 4A-3B+6(C-2B+2A)$

$= 4A-3B+6C-12B+12A = 16A-15B+6C$

b) Let $A=a^2-ab-2b^2$, $B=a^2-2ab+3b^2$, $C=3a^2+4ab-5b^2$. Simplify the following.

$4A-3\big(B-2(C-2(B-A))\big) = 16A-15B+6C$

$= 16(a^2-ab-2b^2)-15(a^2-2ab+3b^2)+6(3a^2+4ab-5b^2)$

$= 19a^2+38ab-107b^2$

2 Expand the following expressions.

a) $(3a-2b)^3 = 27a^3-54a^2b+36ab^2-8b^3$

b) $(5a^2-2bc)(3a^2-4bc) = 15a^4-26a^2bc+8b^2c^2$

c) $(x^2+x+1)(x^2-x+1) = (x^2+1+x)(x^2+1-x)$

$= (x^2+1)^2-x^2 = x^4+x^2+1$

3 Factor the following expressions.

a) $3x^2-4x-4 = (x-2)(3x+2)$

b) $4a^2-12ab+9b^2 = (2a-3b)^2$

c) $18x^2-2y^2 = 2(9x^2-y^2) = 2(3x-y)(3x+y)$

d) $27x^3-y^3 = (3x-y)(9x^2+3xy+y^2)$

4 Find the greatest common divisor (GCD) and the least common multiple (LCM) of each of the following polynomials.

a) a^2b^3, a^4b^4c, $a^3b^2c^2$

GCD= a^2b^2

LCM= $a^4b^4c^2$

b) $(x+1)(x-1)^2$, $(x+1)^2(x+2)(x-1)$

GCD= $(x+1)(x-1)$

LCM= $(x+1)^2(x-1)^2(x+2)$

5 Using long division, find the quotient and the remainder.

$$\begin{array}{r} x^2+x+1 \\ 2x^2-x-1 \;\overline{)\; 2x^4+x^3\qquad -3x-3} \\ \underline{2x^4-x^3-x^2} \\ 2x^3+x^2-3x \\ \underline{2x^3-x^2-x} \\ 2x^2-2x-3 \\ \underline{2x^2-x-1} \\ -x-2 \end{array}$$

Quotient = x^2+x+1 Remainder = $-x-2$

6 Let $P(x)=x^3-5x^2+3x+9$.

a) Find the value $P(-1)$. $P(-1) = -1-5-3+9 = 0$

b) Factor the polynomial $P(x)$.

$P(x) = (x+1)(x^2-6x+9)$

$= (x+1)(x-3)^2$

7 Express the following fraction of polynomials as the sum of a polynomial and a fraction whose numerator has a degree less than its denominator.

a) $\dfrac{3x+1}{x+2} = 3-\dfrac{5}{x+2}$

b) $\dfrac{x^2-x-1}{x-3} = x+2+\dfrac{5}{x-3}$

8 Reduce each of the following fractions to its lowest terms.

a) $\dfrac{(-2x^2y)^3}{(-3x^3y^2)^2} = \dfrac{-8x^6y^3}{9x^6y^4} = \dfrac{-8}{9y}$

b) $\dfrac{2x^2y^2}{6x^2y-4xy^2} = \dfrac{xy}{3x-2y}$

c) $\dfrac{a^3-b^3}{a^3-ab^2} = \dfrac{(a-b)(a^2+ab+b^2)}{a(a-b)(a+b)} = \dfrac{a^2+ab+b^2}{a(a+b)}$

9 Simplify the following expressions.

a) $\dfrac{12x}{\frac{3}{x}} = 12x\times\dfrac{x}{3} = 4x^2$

b) $\left(-\dfrac{2xy^2}{a^2}\right)\div\left(-\dfrac{ay}{4x^2}\right) = -\dfrac{2xy^2}{a^2}\times\left(-\dfrac{4x^2}{ay}\right) = \dfrac{8x^3y}{a^3}$

c) $\dfrac{x^2-x-6}{x^2+4x+4}\div\dfrac{x^2-5x+6}{x^2-2x-8} = \dfrac{(x-3)(x+2)}{(x+2)^2}\times\dfrac{(x-4)(x+2)}{(x-2)(x-3)}$

$= \dfrac{x-4}{x-2}$

d) $\dfrac{a^2-b^2}{(a-b)^2}\div\dfrac{a^2-2ab+b^2}{a^3-a^2b+ab^2}\times\dfrac{a^2b+ab^2}{a^3+b^3}$

$= \dfrac{(a-b)(a+b)}{(a-b)^2}\times\dfrac{a(a^2-ab+b^2)}{(a-b)^2}\times\dfrac{ab(a+b)}{(a+b)(a^2-ab+b^2)}$

$= \dfrac{a^2b(a+b)}{(a-b)^3}$

e) $\dfrac{b-c}{bc}+\dfrac{c-a}{ca}+\dfrac{a-b}{ab} = \dfrac{a(b-c)+b(c-a)+c(a-b)}{abc}$

$= \dfrac{ab-ac+bc-ab+ac-bc}{abc} = 0$

(Continues on the reverse side)

f) $\dfrac{2x^2}{4x^2-y^2}+\dfrac{x-y}{y-2x}=\dfrac{2x^2}{(2x-y)(2x+y)}-\dfrac{x-y}{2x-y}$

$=\dfrac{2x^2-(x-y)(2x+y)}{(2x-y)(2x+y)}=\dfrac{xy+y^2}{(2x-y)(2x+y)}$

$=\dfrac{y(x+y)}{(2x-y)(2x+y)}$

g) $\dfrac{x+2}{2x^2-x-1}+\dfrac{3x+2}{2x^2+3x+1}=\dfrac{x+2}{(2x+1)(x-1)}+\dfrac{3x+2}{(2x+1)(x+1)}$

$=\dfrac{(x+1)(x+2)+(3x+2)(x-1)}{(2x+1)(x-1)(x+1)}$

$=\dfrac{2x(2x+1)}{(2x+1)(x-1)(x+1)}=\dfrac{2x}{(x-1)(x+1)}$

h) $\dfrac{a}{ab-b^2}+\dfrac{b}{ba-a^2}=\dfrac{a}{b(a-b)}+\dfrac{b}{a(b-a)}$

$=\dfrac{a}{b(a-b)}-\dfrac{b}{a(a-b)}=\dfrac{a^2-b^2}{ab(a-b)}$

$=\dfrac{(a-b)(a+b)}{ab(a-b)}=\dfrac{a+b}{ab}$

i) $\dfrac{1}{x(x+1)}+\dfrac{1}{(x+1)(x+2)}+\dfrac{1}{(x+2)(x+3)}$

$=\dfrac{(x+2)(x+3)+x(x+3)+x(x+1)}{x(x+1)(x+2)(x+3)}$

$=\dfrac{3x^2+9x+6}{x(x+1)(x+2)(x+3)}=\dfrac{3(x+1)(x+2)}{x(x+1)(x+2)(x+3)}$

$=\dfrac{3}{x(x+3)}$

j) $\dfrac{1}{c-\dfrac{1}{c+\dfrac{1}{c}}}=\dfrac{1}{c-\dfrac{1}{\dfrac{c^2+1}{c}}}=\dfrac{1}{c-\dfrac{c}{c^2+1}}$

$=\dfrac{1}{\dfrac{c^3+c-c}{c^2+1}}=\dfrac{c^2+1}{c^3}$

k) $\dfrac{2a}{\dfrac{1}{1-\dfrac{1}{a}}-\dfrac{1}{1+\dfrac{1}{a}}}=\dfrac{2a}{\dfrac{1}{\dfrac{a-1}{a}}-\dfrac{1}{\dfrac{a+1}{a}}}$

$=\dfrac{2a}{\dfrac{a}{a-1}-\dfrac{a}{a+1}}=\dfrac{2a}{\dfrac{a(a+1)-a(a-1)}{(a-1)(a+1)}}$

$=\dfrac{2a}{\dfrac{2a}{(a-1)(a+1)}}=(a-1)(a+1)$

10 Solve the following equation with respect to the unknown indicated in [ ].

a) $\dfrac{1}{p}+\dfrac{1}{q}=\dfrac{1}{r}$ [p] $\quad \dfrac{1}{p}=\dfrac{1}{r}-\dfrac{1}{q}$

$\Rightarrow \dfrac{1}{p}=\dfrac{q-r}{rq} \Rightarrow p=\dfrac{qr}{q-r}$

11 Solve each of the following inequalities and express the solution on a number line.

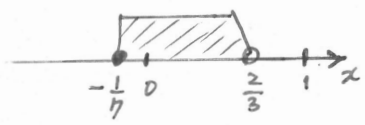
a) $\dfrac{x+4}{3}\geqq 1-2x>\dfrac{x}{2}-\dfrac{2}{3}$

$\begin{cases}\dfrac{x+4}{3}\geqq 1-2x & \cdots ①\\ 1-2x>\dfrac{x}{2}-\dfrac{2}{3} & \cdots ②\end{cases}$

① : $x+4\geqq 3-6x \Rightarrow 7x\geqq -1 \Rightarrow x\geqq -\dfrac{1}{7}$ $\cdots ①'$

② : $6-12x>3x-4 \Rightarrow 15x<10 \Rightarrow x<\dfrac{2}{3}$ $\cdots ②'$

①' and ②' : $-\dfrac{1}{7}\leqq x<\dfrac{2}{3}$



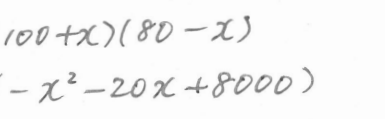
b) $|3x+2|\geqq 5$

$3x+2\leqq -5$ ① or $3x+2\geqq 5$ ②

① $3x\leqq -7 \Rightarrow x\leqq -\dfrac{7}{3}$

② $3x\geqq 3 \Rightarrow x\geqq 1$

$x\leqq -\dfrac{7}{3}$ or $x\geqq 1$



12 A group of people wants to create a team T-shirt for a sports festival. The usual price for a T-shirt is 1,000 yen per T-shirt, but if you pay the 4,000 yen fee, the member's price will be 850 yen per T-shirt. If you become a member, how many T-shirts you must buy to make the total amount cheaper.

Suppose buying x T-shirts

non member : $1000x$ (yen)

member : $4000+850x$ (yen)

$1000x>4000+850x$

$150x>4000$

$x>26.66\ldots$

If you buy 27 T-shirts or more, member price become cheaper.

13 If a product is sold at 100 yen, it makes 800 sales a day and if the price is increased, it is considered that sales will decrease by 100 per price increase of 10 yen. Assuming that sales are linearly related to the price, what is the selling price to maximize sales?

Suppose price is increased x yen.

price : $100+x$ (yen)

Sales : $800-10x$

total sales : $(100+x)(800-10x)$

$=10(100+x)(80-x)$

$=10(-x^2-20x+8000)$

$=-10(x+10)^2+81000$

max at $x=-10$

maximum sales at price 90 yen