1] Find each of the following values.
a) $\log _{2} 32=$
b) $\log _{25} 5=$
c) $\log _{2} \frac{1}{4}=$
d) $\log _{4} 16=$
e) $\log _{5} 5=$
f) $\log _{4} 2=$
g) $\log _{3} \frac{1}{3}=$
h) $\log _{8} \sqrt{2}=$

2 Solve each of the following equations for $s$
a) $\log _{2} x=3$
b) $\log _{4} x=-\frac{1}{2}$
c) $\log _{3} x=2$
d) $\log _{27} x=3$

3] By the definition of $\operatorname{logarithm}, ~_{\log }^{b} \boldsymbol{M}=m$ implies $M=b^{m}$, and $\log _{b} N=n$ implies $N=b^{n}$. Prove each of the following properties of logarithm by setting $M=b^{m}$ and $N=b^{n}$ and using the rules of exponents.
a) $\log _{b}(M \times N)=\log _{b} M+\log _{b} N$
b) $\log _{b}\left(\frac{M}{N}\right)=\log _{b} M-\log _{b} N$

4) Let $p=\log _{a} 2$ and $q=\log _{a} 3$. Express each of the following in terms of $p$ and $q$.
a) $\log _{a} 8=$
b) $\log _{a} 18=$
c) $\log _{a} 12=$
d) $\log _{a} 1.5=$
5) Simplify the following.
a) $\log _{2} \frac{3}{4}-\log _{2} \frac{3}{2}=$
b) $\frac{1}{2} \log _{3} 5-\log _{3} \frac{\sqrt{5}}{3}=$
c) $\log _{2}(3+\sqrt{5})+\log _{2}(3-\sqrt{5})=$
d) $3 \log _{5} 15-\log _{5} 135=$
6) Simplify the following.
a) $\frac{1}{3} \log _{10} 125+\log _{10} \frac{3}{5}-\log _{10} 0.3=$
b) $\log _{a} \frac{A}{B}+\log _{a} \frac{B}{C}+\log _{a} \frac{C}{A}=$

7] By the definition of logarithm, we have $a^{\log _{a} b}=b$. By taking the log base $c$ of both sides of this, express $\log _{a} b$ in terms of $\log _{c} a$ and $\log _{c} b$. [The formula obtained is called the change-of-base formula.]
c) $\log _{b} M^{r}=r \log _{b} M$

8] Assuming $\log _{2} 3=m$, express $\log _{4} 6$ and $\log _{3} 2$ in terms of $m$.
a) $\log _{4} 6=$
b) $\log _{3} 2=$
9. Simplify the following using the change-of-base formula.
a) $\log _{a} b \cdot \log _{b} a=$
b) $\log _{a} b \cdot \log _{b} c \cdot \log _{c} a=$
10. Fill in the blanks in the following table for the function $y=\log _{2} x$ with decimals. Here, we assume $2^{0.5}=1.414$.

| $x$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | -3 | -2.5 | -2 | -1.5 | -1 | -0.5 | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |

11 Using the table above, draw a graph of the exponential function $y=\log _{2} x$ as carefully as possible.


12 Draw the graph of each of the functions $y=3^{x}, y=3^{-x}, y=\log _{3} x$, and $y=-\log _{3} x$.


13 How many digits is $2^{32}$. Use the fact $\log _{10} 2 \fallingdotseq 0.3010$.

14 Every time a ray of light passes through a certain glass, it loses $1 / 10$ of its luminous intensity. How many times this glass is stacked will reduce light intensity to $1 / 3$ or less of the original. Use $\log _{10} 3 \fallingdotseq 0.4771$.

