**1**   Simplify each expression.

a)   $\sqrt[5]{32} =$

b)   $\sqrt[3]{-8} =$

c)   $\sqrt[3]{-0.001} =$

d)   $\sqrt[4]{\dfrac{81}{16}} =$

**2**   Simplify each expression.

a)   $\sqrt[3]{4} \times \sqrt[3]{16} =$

b)   $\sqrt[4]{\dfrac{9}{4}} \times \sqrt[4]{36} =$

c)   $\sqrt[3]{0.0001} \times \sqrt[3]{10} =$

d)   $\sqrt[4]{80} \div \sqrt[4]{5} =$

**3**   Simplify each expression.

a)   $\left(\sqrt[6]{4}\right)^3 =$

b)   $\sqrt[3]{-\sqrt{729}} =$

**4**   Simplify each expression.

a)   $3^0 =$

b)   $0.1^{-1} =$

c)   $(2.5^0)^{-4} =$

d)   $\left(\dfrac{2}{3}\right)^{-3} =$

**5**   Simplify each expression.

a)   $27^{-\frac{1}{3}} =$

b)   $8^{-\frac{2}{3}} =$

c)   $32^{0.4} =$

d)   $100^{-\frac{1}{2}} =$

**6**   Express each of the following expression with a rational exponent.   Here we assume $a > 0$.

a)   $\sqrt[5]{a^7} =$

b)   $\dfrac{1}{\sqrt[4]{a}} =$

c)   $\sqrt[3]{a}\sqrt{a} =$

d)   $\sqrt{\sqrt[3]{a}} =$

**7**   Express each of the following in the form $\sqrt[n]{a^m}$. Here we assume $a > 0$.

a)   $a^{\frac{2}{3}} =$

b)   $a^{1.75} =$

c)   $\dfrac{1}{a^{-2.5}} =$

d)   $a^{-3.6} \times a^{4.3} =$

**8**   Simplify each of the following, and express it with a rational exponent. Here, we assume $a > 0$, $b > 0$.

a)   $\dfrac{a\sqrt[3]{a}}{\sqrt[6]{a}} =$

b)   $\sqrt[4]{a^3} \times \sqrt[6]{a^4} =$

c)   $\sqrt[3]{a} \div \sqrt[4]{a^3} =$

d)   $\dfrac{\sqrt{a^3 b} \times \sqrt[3]{ab^2}}{\sqrt[6]{a^5 b}} =$

e)   $10^{-\frac{1}{6}} \div 10^{-\frac{1}{3}} \times 10^{\frac{5}{6}} =$

f)   $a^{0.4} \div a^{-\frac{1}{3}} =$

g)   $a^{-\frac{1}{2}} \times a^{\frac{1}{3}} \div a^{\frac{5}{6}} =$

h)   $(a^{p-q})^r (a^{q-r})^p (a^{r-p})^q =$

**9**   Simplify each expression.

a)   $\left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)^2 =$

b)   $(ab^{-1} + a^{-1}b)^2 - (ab^{-1} - a^{-1}b)^2 =$

**10** a) Suppose $2^{0.5} \fallingdotseq 1.414$. Then, the approximate value of $2^{-0.5}$ can be calculated as $2^{-0.5} = 2^{0.5} \times 2^{-1} \fallingdotseq 1.414 \div 2 = 0.707$. Apply this to find the approximate value of $2^{-1.5}$, $2^{1.5}$, ..., and fill in the blanks in the following table for the function $y = 2^x$ with <u>decimals</u>.

| $x$ | $-3$ | $-2.5$ | $-2$ | $-1.5$ | $-1$ | $-0.5$ | $0$ | $0.5$ | $1$ | $1.5$ | $2$ | $2.5$ | $3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | | | | | | | |

b) Using the table above, draw a graph of the exponential function $y = 2^x$ as <u>as carefully as possible</u>.
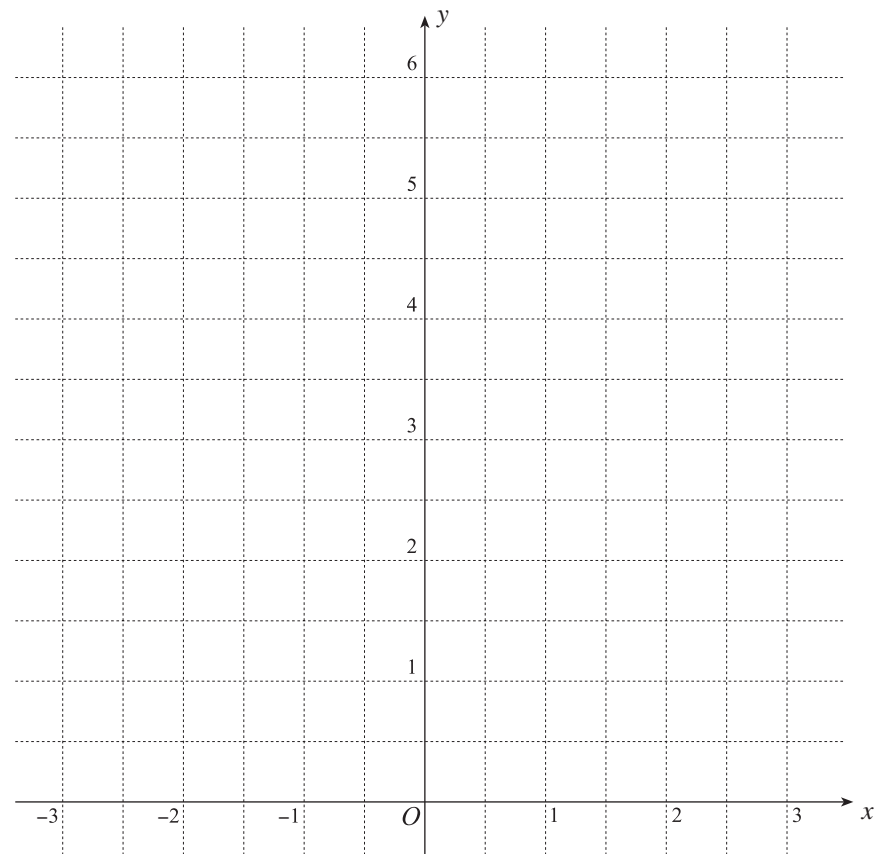


**11** Solve each equation.

a) $4^x = 32$

b) $125^x = 25$

c) $3^x = 1$

d) $3^x = \sqrt{27}$

**12** Arrange the following numbers in ascending order. $10^{-1}$, $10^{\frac{3}{2}}$, $10^0$, $10^{-\frac{1}{2}}$, $10^{\frac{2}{3}}$

**13** To compare $\sqrt{2}$ with $\sqrt[3]{3}$, represent both by the same power root, such as $\sqrt{2} = \sqrt[6]{8}$, $\sqrt[3]{3} = \sqrt[6]{9}$, and then compare the numbers within $\sqrt[6]{\phantom{x}}$. Arrange each of the following numbers in ascending order.

a) $\sqrt{6}$, $\sqrt[3]{14}$

b) $\sqrt{10}$, $\sqrt[3]{31}$

c) $\sqrt[3]{3}$, $\sqrt[4]{4}$, $\sqrt[5]{5}$

**14** Draw the graph of functions $y = 3^x$, $y = 3^{-x}$, $y = -3^x$, and $y = -3^{-x}$.



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