Student ID No.	Name
1 9 F 1 1	

1 For each of the following parabolas, find the vertex and sketch its graph carefully in the coordinate plane below. (Pay particular attention to the vertex, the intersection with the x axis, and so on.)

a) $y = x^2 + 6x + 5$

$= (x+3)^2 - 4$

vertex $(-3, -4)$

b) $y = 2x^2 - 8x + 9$

$= 2(x-2)^2 + 1$

vertex $(2, 1)$

c) $y = -x^2 + 5x - 6$

$= -(x-\frac{5}{2})^2 + \frac{1}{4}$

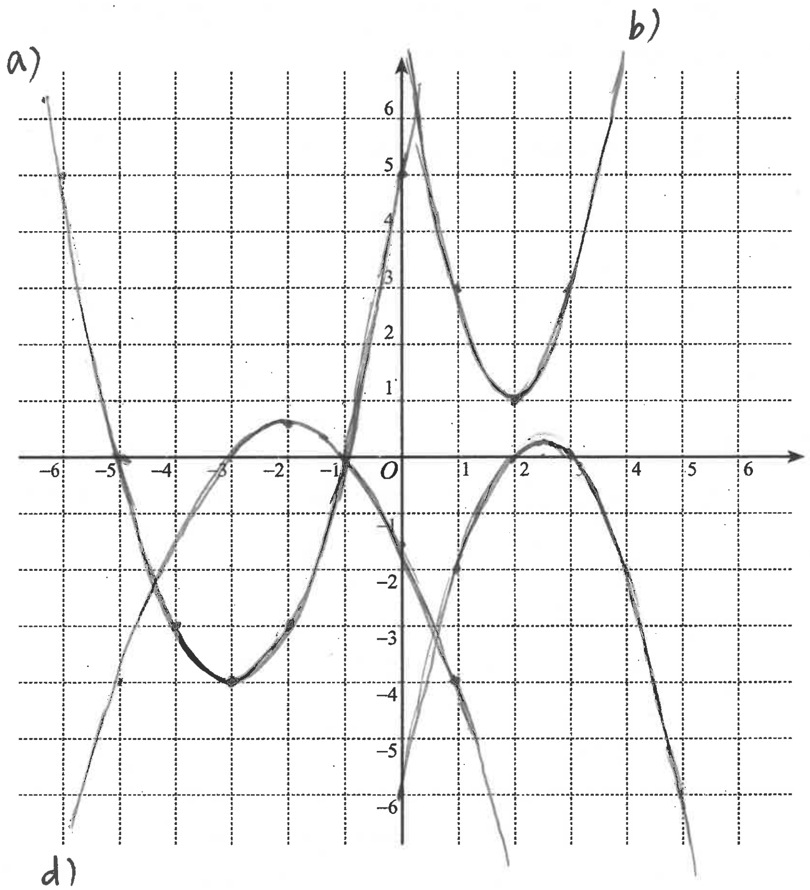
vertex $(\frac{5}{2}, \frac{1}{4})$

d) $y = -\frac{3}{2} - 2x - \frac{1}{2}x^2$

$= -\frac{1}{2}(x^2+4x+3)$

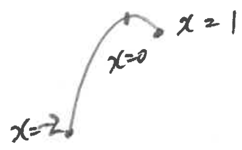
$= -\frac{1}{2}(x+2)^2 + \frac{1}{2}$

vertex $(-2, \frac{1}{2})$



2 For each of the following functions, find the maximum and minimum values when x varies within the domain indicated in ( ). Also, find the value of x at which the function attains its maximum and minimum.

c) $y = 3 - x^2 \quad (-2 \leqq x \leqq 1)$

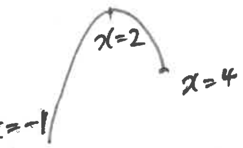


max. 3 $(x=0)$

min. -1 $(x=-2)$

d) $y = -x^2 + 4x \quad (-1 \leqq x \leqq 4)$

$= -(x-2)^2 + 4$

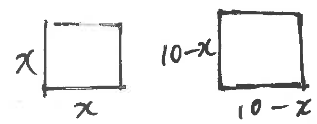


max. 4 $(x=2)$

min. -5 $(x=-1)$

3 Cut a 40 cm long wire into two pieces, and fold each of the two wires to make two squares. How do I cut the wires to minimize the sum of the areas of those squares? Also, find the minimum value of the sum of areas.

$4x$: length of one piece. → the other $40-4x = 4(10-x)$



Area $= x^2 + (10-x)^2$

$= 2x^2 - 20x + 100$

$= 2(x-5)^2 + 50$

min. at $x=5$. minimum $= 50$

Ans cut into half. 50cm^2

4 A local newspaper currently has 84,000 subscribers at a quarterly charge of \$30. Market research has suggested that if the owners raise the price by \$2, they would lose 5,000 subscribers. Assuming that subscriptions are linearly related to the price, what price should the newspaper charge for a quarterly subscription to maximize their revenue?

price ↑ \$2 ⇒ subscribers ↓ 5000 means

— ↑ \$x ⇒ — ↓ $5000 \times \frac{x}{2}$

Revenue $= \underbrace{(30+x)}_{\text{price}}\underbrace{(84000-2500x)}_{\text{subscribers}} = -100(25x-840)(x+30)$

$= -100(25x^2 - 90x - 25200) = -2500(x-\frac{9}{5})^2 + 2528100$

max at $x = \frac{9}{5}$ ⇒ at \$31.8

(continues on the reverse side)

5 Solve the following equations over the complex numbers.

a) $2x^2+7x+3=0$

$(2x+1)(x+3)=0$

$x=-\frac{1}{2}, -3$

b) $4x^2-12x+9=0$

$(2x-3)^2=0$

$x=\frac{3}{2}$ (double solution)

c) $x^2+3x-2=0$

$x=\frac{-3\pm\sqrt{17}}{2}$

d) $3x^2-5x-2=0$

$(3x+1)(x-2)=0$

$x=-\frac{1}{3}, 2$

e) $x^2-2x+5=0$

$x=1\pm\sqrt{-4}$

$=1\pm 2i$

f) $\frac{x^2}{3}+\frac{x}{2}-\frac{1}{4}=0$

$4x^2+6x-3=0$

$x=\frac{-3\pm\sqrt{21}}{4}$

6 If the quadratic equation $x^2+mx-m+3=0$ has a double solution, find the value of the constant m. Also, find its solution.

double solution $\Leftrightarrow D=0$

$D=m^2-4(-m+3)=0$

$m^2+4m-12=0$

$(m-2)(m+6)=0$

$\therefore m=2, -6$

i) $m=2$

$x^2+2x+1=0$

$(x+1)^2=0$ $\quad\therefore x=-1$

ii) $m=-6$

$x^2-6x+9=0$

$(x-3)^2=0$ $\quad\therefore x=3$

7 A person has a garden that has a length 10 m longer than the width. Set up a quadratic equation to find the dimensions of the garden if its area is 119 m^2. Solve the quadratic equation to find the length and width.

x : width. $\rightarrow$ length $x+10$

Area $=x(x+10)$

$x(x+10)=119$

$x^2+10x-119=0$

$(x-7)(x+17)=0$

since $x>0$, $x=7$ (m)

$\begin{cases}\text{length} & 17\,\text{m}\\ \text{width} & 7\,\text{m}\end{cases}$

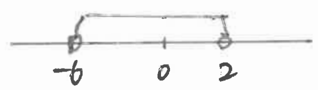
Masato Kuwata(kuwata@tamacc.chuo-u.ac.jp)

8 次の不等式を解け．またその解を数直線上に表せ．

a) $x^2+4x-12<0$

$(x-2)(x+6)<0$

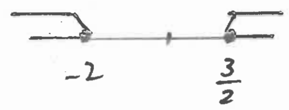
$-6<x<2$



b) $2x^2+x-6\geqq 0$

$(2x-3)(x+2)\geqq 0$

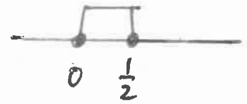
$x\leqq -2,\ x\geqq\frac{3}{2}$



c) $2x^2-x\leqq 0$

$x(2x-1)\leqq 0$

$0\leqq x\leqq\frac{1}{2}$

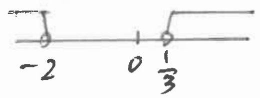


d) $6x^2+10x-4>0$

$3x^2+5x-2>0$

$(3x-1)(x+2)>0$

$x<-2,\ x>\frac{1}{3}$



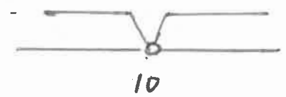
e) $x(x-8)>12x-100$

$x^2-8x>12x-100$

$x^2-20x+100>0$

$(x-10)^2>0$

$x<10,\ x>10\ (x\neq 10)$



f) $x^2-x+1\leqq 5x-8$

$x^2-6x+9\leqq 0$

$(x-3)^2\leqq 0$

$x\leqq 3,\ x\geqq 3\ (x=3)$

9 For a rectangle of 20 cm in perimeter, in order to make its area more than or equal to 15 cm^2 and less than 20cm^2, what the length of the longer side of the rectangle should be?

[Hint: If the length of the longer side is x, the length of the shorter side is $10-x$. You must also consider the condition that x is greater than $10-x$.]

x : length of the longer side $\rightarrow$ shorter side : $10-x$

$x>10-x \Rightarrow x>5$ ⋯①

Area $=x(10-x)$ $\rightarrow$ $15\leqq x(10-x)<20$

$15\leqq x(10-x)$

$x^2-10x+15\leqq 0$

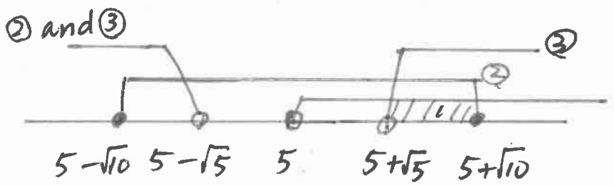
$(x-5-\sqrt{10})(x-5+\sqrt{10})\leqq 0 \Rightarrow +5-\sqrt{10}\leqq x\leqq 5+\sqrt{10}$ ⋯②

$x(10-x)<20$

$x^2-10x+20>0$

$(x-5-\sqrt{5})(x-5+\sqrt{5})>0 \Rightarrow x<5-\sqrt{5},\ x>5+\sqrt{5}$ ⋯③

① and ② and ③



Ans $5+\sqrt{5}<x\leqq x+\sqrt{10}$