Student ID No.										Name
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1 Factor the following polynomials.

a) $3ab - 6ac =$

b) $2a^2b - ab^2 =$

c) $x^2 - x =$

d) $(a+b)x - (a+b)y =$

e) $x^2 + x - 6 =$

f) $x^2 - 7x + 10 =$

g) $3x^2 - 18x + 27 =$

h) $x^2 - 11xy + 24y^2 =$

i) $25x^2 - 4 =$

j) $x^3 + 8 =$

k) $x^4 + x =$

l) $3x^2 - 5x - 2 =$

2 Using long division, find the quotient and the remainder. (Here, a is a constant.)

a)

$x - 2 \overline{)\, x^3 - 2x^2 + 4x - 8}$

Quotient =

Remainder =

b)

$x^2 - 3x + 2 \overline{)\, x^3 \qquad - 9x + 8}$

Quotient =

Remainder =

c)

$2x^2 \quad - 1 \overline{)\, x^3 - 3x^2 \qquad + 4}$

Quotient =

Remainder =

d)

$x^2 + ax - a^2 \overline{)\, x^3 - ax^2 - 3a^2x}$

Quotient =

Remainder =

3 Consider the polynomial $f(x) = x^3 - 3x + 1$.

a) Divide $f(x)$ by $x - 2$, and find the quotient and the remainder.

$x - 2 \overline{)\, x^3 \qquad - 3x + 1}$

b) Compute the value $f(2)$, and show that it agrees with the result of a).

(continues on the reverse side)

4 a) Using the remainder theorem, find the remainder when $f(x) = x^3 - 3x^2 + 4$ is divided by each of the following.

1) $x - 1$ 2) $x - 2$

3) $x + 1$ 4) $x + 2$

b) Which of $x - 1$, $x - 2$, $x + 1$, and $x + 2$ are the factors of $x^3 - 3x^2 + 4$.

c) Factor the polynomial $x^3 - 3x^2 + 4$.

5 Let $P(x) = x^3 - x^2 - 5x - 3$.

a) Find the value $P(-1)$.

b) Factor $P(x)$.

6 Among the dividend, the divisor, the quotient, and the remainder, there is a relation

$$(\text{Dividend}) = (\text{Divisor}) \times (\text{Quotient}) + (\text{Remainder})$$

Express it in the fractional form, we can express it

$$\frac{(\text{Dividend})}{(\text{Divisor})} = (\text{Quotient}) + \frac{(\text{Remainder})}{(\text{Divisor})}$$

For example, if we divide 17 by 7, the quotient is 2, and the remainder is 3. Thus, we have $\frac{17}{7} = 2 + \frac{3}{7}$. Similarly, if we divide $2x^2 - 5x + 1$ by $x - 2$, then the quotient is $2x - 1$, and the remainder is -1. Thus, we can write $\frac{2x^2 - 5x + 1}{x - 2} = 2x - 1 + \frac{-1}{x - 2}$. In other word, a fraction of polynomials can be expressed by the sum of polynomials and a fraction whose numerator has a degree lower than its denominator.

Express the following fractions of polynomials can be expressed by the sum of polynomials and a fraction whose numerator has a degree lower than its denominator.

a) $\frac{5x - 3}{x - 2} =$

b) $\frac{2x^2 - x + 3}{2x + 1} =$

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