

$$\text{a)} \begin{pmatrix} -\frac{4}{5} & \frac{2}{5} & \frac{1}{2} \\ \frac{2}{5} & -\frac{4}{5} & \frac{3}{10} \\ \frac{2}{5} & \frac{2}{5} & -\frac{4}{5} \end{pmatrix} \begin{array}{l} \textcircled{1} \times (-\frac{5}{4}) \\ \textcircled{2} \times 5 \\ \textcircled{3} \times 5 \end{array} \longrightarrow \begin{pmatrix} \textcircled{1} & -\frac{1}{2} & -\frac{5}{8} \\ 2 & -4 & \frac{3}{2} \\ 2 & 2 & -4 \end{pmatrix} \begin{array}{l} \textcircled{2} - \textcircled{1} \times 2 \\ \textcircled{3} - \textcircled{1} \times 2 \end{array} \longrightarrow \begin{pmatrix} \textcircled{1} & -\frac{1}{2} & -\frac{5}{8} \\ 0 & -3 & \frac{11}{4} \\ 0 & 3 & -\frac{11}{4} \end{pmatrix}$$

$$\begin{array}{l} \textcircled{2} \times (-\frac{1}{3}) \\ \longrightarrow \end{array} \begin{pmatrix} \textcircled{1} & -\frac{1}{2} & -\frac{5}{8} \\ 0 & \textcircled{1} & -\frac{11}{12} \\ 0 & 3 & -\frac{11}{4} \end{pmatrix} \begin{array}{l} \textcircled{3} - \textcircled{2} \times 3 \\ \textcircled{1} + \textcircled{2} \times \frac{1}{2} \end{array} \longrightarrow \begin{pmatrix} \textcircled{1} & 0 & -\frac{13}{12} \\ 0 & \textcircled{1} & -\frac{11}{12} \\ 0 & 0 & 0 \end{pmatrix}$$

$$z = 12t \text{ とおくと } x = 13t, y = 12t$$

$$\therefore \begin{cases} x = 13t \\ y = 11t \\ z = 12t \end{cases} \quad t \text{ は任意の実数}$$

b) ジャガイモの値段 x , トマトの値段 y , レタスの値段 z とする

$$\begin{cases} \frac{1}{5}x + \frac{2}{5}y + \frac{1}{2}z = x \\ \frac{2}{5}x + \frac{1}{5}y + \frac{3}{10}z = y \\ \frac{2}{5}x + \frac{2}{5}y + \frac{1}{5}z = z \end{cases} \Leftrightarrow \begin{cases} -\frac{4}{5}x + \frac{2}{5}y + \frac{1}{2}z = 0 \\ \frac{2}{5}x - \frac{4}{5}y + \frac{3}{10}z = 0 \\ \frac{2}{5}x + \frac{2}{5}y - \frac{4}{5}z = 0 \end{cases}$$

$$\text{a) より } x = 13t, y = 11t, z = 12t$$

$$11t = 9900 \text{ とおきように } t = 900 \text{ とおくと } x = 11700 \text{円}, y = 9900 \text{円}, z = 10800 \text{円}$$

$$\text{c)} \begin{pmatrix} p_{n+1} \\ s_{n+1} \\ g_{n+1} \end{pmatrix} = \begin{pmatrix} \frac{1}{5} & \frac{2}{5} & \frac{1}{2} \\ \frac{2}{5} & \frac{1}{5} & \frac{3}{10} \\ \frac{2}{5} & \frac{2}{5} & \frac{1}{5} \end{pmatrix} \begin{pmatrix} p_n \\ s_n \\ g_n \end{pmatrix}$$

不動ベクトル $\begin{pmatrix} p \\ g \\ r \end{pmatrix}$ とおくと p, g, r は a) と同じ方程式をみたす。

$$\therefore p : g : r = 13 : 11 : 12$$

$$\text{[2] a)} \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ -1 & 2 & 2 & 0 & 1 & 0 \\ -2 & -1 & -1 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} \textcircled{2} + \textcircled{1} \\ \textcircled{3} + \textcircled{1} \times 2 \end{array} \longrightarrow \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & -3 & -4 & 2 & 0 & 1 \end{array} \right)$$

$$\begin{array}{l} \textcircled{1} + \textcircled{2} \\ \textcircled{3} + \textcircled{2} \times 3 \end{array} \longrightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 5 & 3 & 1 \end{array} \right) \begin{array}{l} \textcircled{3} \times (-1) \end{array} \longrightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -5 & -3 & -1 \end{array} \right)$$

$$\textcircled{2}-\textcircled{3} \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 0 \\ 0 & 1 & 0 & 6 & 4 & 1 \\ 0 & 0 & 1 & -5 & -3 & -1 \end{array} \right) \quad \therefore A^{-1} = \begin{pmatrix} 2 & 1 & 0 \\ 6 & 4 & 1 \\ -5 & -3 & -1 \end{pmatrix}$$

$$b) \quad A \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \quad \text{よって} \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 0 \\ 6 & 4 & 1 \\ -5 & -3 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \\ 2 \end{pmatrix}$$

$$x = -1, y = -4, z = 2$$

$$\textcircled{3} \quad PQ = \begin{pmatrix} 5 & 2 & 5 \\ -4 & 4 & 3 \\ 3 & 2 & 4 \end{pmatrix} \quad QP = \begin{pmatrix} 8 & -1 \\ 3 & 5 \end{pmatrix}$$

$$\textcircled{4} \quad a) \quad A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad B = \begin{pmatrix} \cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & \cos 90^\circ \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$b) \quad ABA = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} \cos(-90^\circ) & -\sin(-90^\circ) \\ \sin(-90^\circ) & \cos(-90^\circ) \end{pmatrix}$$

$\therefore (x, y) \mapsto (x', y')$ は -90° 回転

$$c) \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \\ = \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{pmatrix} \quad \text{よって} \quad -\theta \text{ 回転}$$