

5 次関数を変数  $x$  で微分せよ。

a)  $f(x) = \frac{4x-1}{x^2+3}$

$$f'(x) = \frac{(4x-1)'(x^2+3) - (4x-1)(x^2+3)'}{(x^2+3)^2}$$

$$= \frac{4x^2+12 - (4x-1) \times 2x}{(x^2+3)^2}$$

$$= \frac{-4x^2 + 2x + 12}{(x^2+3)^2}$$

$$\left( = \frac{-2(2x+3)(x-2)}{(x^2+3)^2} \right)$$

c)  $f(x) = \frac{4-x^2}{x^2-2x+3}$

$$f'(x) = \frac{-2x(x^2-2x+3) - (4-x^2)(2x-2)}{(x^2-2x+3)^2}$$

$$= \frac{-2x^3 + 4x^2 - 6x + 2x^3 - 2x^2 - 8x + 8}{(x^2-2x+3)^2}$$

$$= \frac{2x^2 - 14x + 8}{(x^2-2x+3)^2}$$

$$= \frac{2(x^2-7x+8)}{(x^2-2x+3)^2}$$

e)  $f(x) = (2x^2+3x-4)^3$

$$f'(x) = 3(2x^2+3x-4)^2(2x^2+3x-4)'$$

$$= 3(4x+3)(2x^2+3x-4)^2$$

b)  $f(x) = \frac{x^2+2}{x^3-4}$

$$f'(x) = \frac{(x^2+2)'(x^3-4) - (x^2+2)(x^3-4)'}{(x^3-4)^2}$$

$$= \frac{2x(x^3-4) - (x^2+2) \times 3x^2}{(x^3-4)^2}$$

$$= \frac{-x^4 - 6x^2 - 8x}{(x^3-4)^2}$$

$$= \frac{-x(x^3+6x+8)}{(x^3-4)^2}$$

d)  $f(x) = x^2 - \frac{3}{x-2}$

$$f'(x) = 2x + \frac{3}{(x-2)^2}$$

f)  $f(x) = \left(\frac{x^3}{3} + 2x - 5\right)^4$

$$f'(x) = 4\left(\frac{x^3}{3} + 2x - 5\right)^3 \left(\frac{x^3}{3} + 2x - 5\right)'$$

$$= 4(x^2+2)\left(\frac{x^3}{3} + 2x - 5\right)^3$$

g)  $f(x) = \sqrt{x^3} + \sqrt[3]{x^2} = x^{\frac{3}{2}} + x^{\frac{2}{3}}$

$$f'(x) = \frac{3}{2}x^{\frac{1}{2}} + \frac{2}{3}x^{-\frac{1}{3}}$$

$$= \frac{3}{2}\sqrt{x} + \frac{2}{3}\frac{1}{\sqrt[3]{x}}$$

i)  $f(x) = (x+1)\sqrt{2-x}$

$$f'(x) = (x+1)'\sqrt{2-x} + (x+1)(\sqrt{2-x})'$$

$$= \sqrt{2-x} + (x+1) \frac{1}{2}(2-x)^{-\frac{1}{2}}(2-x)'$$

$$= \sqrt{2-x} - \frac{x+1}{2\sqrt{2-x}}$$

$$\left( = \frac{2(2-x) - x + 1}{2\sqrt{2-x}} \right)$$

$$= \frac{-3(x-1)}{2\sqrt{2-x}}$$

k)  $f(x) = \frac{1}{\sqrt{x^2-2x}} = (x^2-2x)^{-\frac{1}{2}}$

$$f'(x) = -\frac{1}{2}(x^2-2x)^{-\frac{3}{2}} \cdot (x^2-2x)'$$

$$= -(x-1)(x^2-2x)^{-\frac{3}{2}}$$

$$= \frac{1-x}{(\sqrt{x^2-2x})^3}$$

h)  $f(x) = \sqrt[4]{(x^2+x+1)^5} = (x^2+x+1)^{\frac{5}{4}}$

$$f'(x) = \frac{5}{4}(x^2+x+1)^{\frac{1}{4}}(x^2+x+1)'$$

$$= \frac{5}{4}(2x+1)\sqrt[4]{x^2+x+1}$$

j)  $f(x) = \sqrt[3]{\frac{x+1}{x+4}} = \left(\frac{x+1}{x+4}\right)^{\frac{1}{3}} = \left(1 - \frac{3}{x+4}\right)^{\frac{1}{3}}$

$$f'(x) = \frac{1}{3}\left(\frac{x+1}{x+4}\right)^{-\frac{2}{3}}\left(1 - \frac{3}{x+4}\right)'$$

$$= \frac{1}{3}\left(\frac{x+1}{x+4}\right)^{-\frac{2}{3}} \times \frac{3}{(x+4)^2}$$

$$= \frac{1}{\sqrt[3]{(x+1)^2} \cdot (x+4)^2}$$

$$= \frac{1}{\sqrt[3]{(x+1)^2(x+4)^2}}$$

l)  $f(x) = \frac{x^2-4x+3}{\sqrt{x}} = x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}$

$$f'(x) = \frac{3}{2}x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}$$

$$= \frac{3}{2}\sqrt{x} - \frac{2}{\sqrt{x}} - \frac{3}{2(\sqrt{x})^3}$$

$$= \frac{3x^2 - 4x - 3}{2x\sqrt{x}}$$