学生証番号 : ________________　氏名 : ____________________

1 行列 P, Q, R, S を次のようにおく．これらの組み合わせのうち，積が定義できる場合すべてについて，その積を計算せよ．

$$P = \begin{pmatrix} -2 & 1 & -1 \end{pmatrix}, \quad Q = \begin{pmatrix} 1 & 1 \\ 2 & -1 \\ -1 & 2 \end{pmatrix}, \quad R = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}, \quad S = \begin{pmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{pmatrix}$$

2 Initially, three firms A, B, and C (numbered 1, 2, and 3) share the market for a certain commodity. Firm A has 20% of the market, B has 60%, and C has 20%. In the course of the next year, the following changes occur:

$$\begin{cases} \text{A keeps 85\% of its customers, while losing 5\% to B, and 10\% to C} \\ \text{B keeps 55\% of its customers, while losing 10\% to A, and 35\% to C} \\ \text{C keeps 85\% of its customers, while losing 10\% to A, and 5\% to B} \end{cases}$$

We can represent market shares of the three firms by means of a *market share vector*, defined as a column vectors $\vec{s}$ whose components are all nonnegative and sum to 1. Define the matrix T and the initial market share vector $\vec{s}$ by

$$T = \begin{pmatrix} 0.85 & 0.10 & 0.10 \\ 0.05 & 0.55 & 0.05 \\ 0.10 & 0.35 & 0.85 \end{pmatrix} \quad \text{and} \quad \vec{s} = \begin{pmatrix} 0.2 \\ 0.6 \\ 0.2 \end{pmatrix}$$

Notice that t_{ij} is the percentage of j's customers who become i's coustomers in the next period. So, T is called the *transition matix*.

a) Compute the vectot $T\vec{s}$.

b) Show that it is also a market share vector.

c) What is the interpretation of $T(T\vec{s})$, $T(T(T\vec{s}))$,...?

【裏に続く】

3 a) $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$，$B = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ とする．AB および BA を求めよ．

b) $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ とする．a) を利用して $ad - bc \neq 0$ のとき $PA = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ となる行列 $P = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ を求めよ．また，このとき $AP = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ となることを確かめよ．

c) $A = \begin{pmatrix} 2 & -5 \\ 3 & 4 \end{pmatrix}$ とする．$PA = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ となる行列 $P = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ を求めよ．

d) $A = \begin{pmatrix} 2 & -5 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ の両辺に b) で求めた P を左から掛けることにより，$\begin{pmatrix} x \\ y \end{pmatrix}$ を求めよ．

e) $\begin{cases} 2x - 5y = -2 \\ 3x + 4y = 3 \end{cases}$ を解け．

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