13. 前期の復習

- キーワード：
  連立 1 次方程式の掃き出し法による解法，行列の基本変形，ピボット，階数，解の自由度，
  行列の乗法，逆行列，交代積，行列式，余因子展開.

1　次の各々の連立 1 次方程式を Gauss の消去法を用いて解け.

a) $\begin{cases} x - y + z + 2w = 1 \\ 3x - 5y - 3z + 4w = -3 \\ 2x - 2y + 5z + w = 5 \end{cases}$　　b) $\begin{cases} x - 3y + z - 2w = 1 \\ -2x + 7y + 8w = -2 \\ -2y - 3z - 6w = 1 \\ -3y - 4z - 9w = -1 \end{cases}$

2　次の各々の連立方程式が解を持つように定数 a を決め，そのときの解をすべて求めよ.

a) $\begin{cases} x + 3y + z - 2w = -2 \\ 2x + 7y - z - 6w = -3 \\ x + 2y + 7z + 3w = -6 \\ 4x + 9y + 7z - 8w = a \end{cases}$　　b) $\begin{cases} x + 3y + 2z + w = 1 \\ 2x + 7y + 5z + 4w = 2 \\ 2x + 9y + 8z + 6w = 6 \\ x + 5y + 5z + 3w = a \end{cases}$

3　次の行列 A について A^2, A^3, A^6 を求めよ.

a) $A = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$　　b) $A = \begin{pmatrix} 4 & -7 \\ 3 & -5 \end{pmatrix}$　　c) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

4　$A = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$，$B = \begin{pmatrix} -1 & 1 \end{pmatrix}$，$C = \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}$，$D = \begin{pmatrix} 0 & -1 \end{pmatrix}$ のとき，これらの行列全部を 1 回ずつ用いて積を作りたい．積が定義できるすべての場合について，A，B，C，D の順とその結果を示せ.

5　次の各々の行列 A について，逆行列 A^{-1} を求め，AA^{-1}，$A^{-1}A$ がともに単位行列になることを確かめよ.

a) $A = \begin{pmatrix} 5 & -3 \\ -6 & 4 \end{pmatrix}$　　b) $A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{pmatrix}$

6　次の連立一次方程式の解を前問で求めた逆行列を用いて求めよ.

a) $\begin{cases} 5x - 3y = -3 \\ -6x + 4y = 2 \end{cases}$　　b) $\begin{cases} x + y + 2z = 3 \\ x + 2y + z = -4 \\ 2x + y + z = 1 \end{cases}$

【裏につづく】

7 行列 $\begin{pmatrix} 2 & 1 \\ 3 & -2 \end{pmatrix}$ で表される 1 次変換により，2 点 A(3, 0)，B(1, 2) がそれぞれ 2 点 C，D に移るとする．このとき，O を原点として，△OCD の面積は △OAB の何倍か．

8 次の各々の行列式をもとめよ．

a) $\begin{vmatrix} 4 & -3 \\ -7 & 5 \end{vmatrix}$

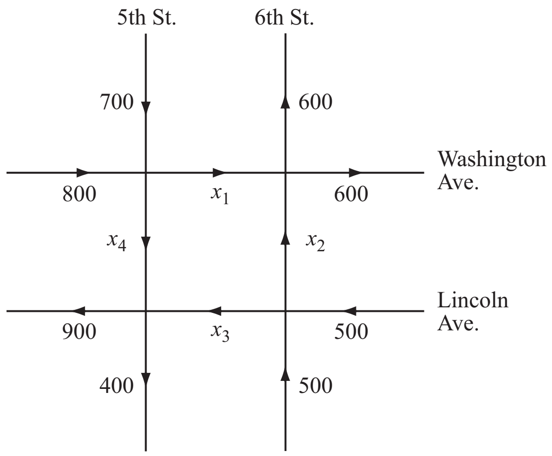
b) $\begin{vmatrix} 1 & -2 & -2 \\ -2 & 3 & -3 \\ 3 & -2 & 3 \end{vmatrix}$

c) $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$

d) $\begin{vmatrix} -5 & 0 & 9 & 7 \\ 2 & 0 & -1 & 0 \\ 8 & 3 & 4 & -6 \\ -4 & 0 & 7 & 0 \end{vmatrix}$

e) $\begin{vmatrix} 1 & -3 & 1 & 1 \\ -1 & 2 & 1 & 2 \\ -2 & 2 & 3 & 1 \\ -1 & 4 & -1 & -4 \end{vmatrix}$

9 *Traffic flow.* The rush-hour traffic flow for a network of four one-way streets in a city is shown in the figure. The numbers next to each street indicate the number of vehicles per hour that enter and leave the network on that street. The variable x_1, x_2, x_3, and x_4 represent the flow of traffic between the four intersections in the network.



a) For a smooth traffic flow, the number of vehicles entering each intersection should always equal the number leaving. For example, since 1500 vehicles enter the intersection of 5th Street and Washington Avenue each hour and $x_1 + x_4$ vehicles leave this intersection, we see that $x_1 + x_4 = 1500$. Find the equations determined by the traffic flow at each of the other three intersections.

b) Find the solution to the system in a).

c) What is the maximum number of vehicles that can travel from Washington Avenue to Lincoln Avenue on 5th Street? What is the minimum number?

d) If traffic lights are adjusted so that 1000 vehicles per hour travel from Washington Avenue to Lincoln Avenue on 5th Stree, determine the flow around the rest of the network.

2012 年 7 月 19 日